

# Universal Coefficient of Performance at Maximum Figure of Merit for Tight-Coupling Refrigerators

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We find and prove that for a refrigerator tightly and symmetrically coupled with two heat baths at temperatures  $T_c$  and  $T_h$  ( $> T_c$ ), the coefficient of performance at maximum figure of merit asymptotically approaches to  $\sqrt{\varepsilon_C}$  when the relative temperature difference between two heat baths  $\varepsilon_C^{-1} \equiv (T_h - T_c)/T_c$  is small enough. This universal coefficient of performance at maximum figure of merit can be regarded as the counterpart of universal efficiency at maximum power output for tight-coupling heat engines operating between two heat baths at small temperature difference in the presence of left-right symmetry.

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*Introduction.*—One of key topics in finite-time thermodynamics is the efficiency at maximum power (EMP) for heat engines. Since the pioneer work made by Curzon and Ahlborn [1], this topic has been fully investigated by many researchers [2–20]. Recent researches are mainly focused on two key issues: One is the universal EMP for tight-coupling heat engines operating between two baths at small temperature difference [6–12]; the other is the global bounds of EMP for heat engines operating between two baths with arbitrary temperature difference [13–20]. In particular, Van den Broeck [6] found that the universal EMP for tight-coupling heat engines was equal to  $\eta_C/2$  up to the first order term of relative temperature difference between two baths, where  $\eta_C$  is the Carnot efficiency which can also be understood as the relative temperature difference for heat engines. The universality up to the second order term of  $\eta_C$  was first observed in Ref. [8], and then proposed as a conjecture in Ref. [9], and finally proved by Esposito *et al.* for tight-coupling heat engines in the presence of left-right symmetry [10]. In addition, Esposito *et al.* also found the EMP of heat engines to be bounded between  $\eta_C/2$  and  $\eta_C/(2 - \eta_C)$  under low-dissipation conditions [13]. It is interesting that these two bounds are also shared by the linear-irreversible heat engines [15]. The accessibility of the bounds for different kinds of heat engines are also fully investigated [16–20]. These researches highly enhance our understanding of the applicable scope of the Curzon-Ahlborn efficiency  $\eta_{CA} \equiv 1 - \sqrt{1 - \eta_C}$  for endoreversible heat engines.

On the other hand, the optimal performance of refrigerators has also attracted much attention [21–33]. Since the power input is not an appropriate optimal target function for Carnot-like refrigerators, it is relatively difficult to define an optimal criterion and obtain its corresponding coefficient of performance (COP) for refrigerators as researchers did for heat engines. Velasco *et al.* [29] adopted the per-unit-time COP as a target function and proved  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1$  to be the upper bound of COP for endoreversible refrigerators operating at the maximum per-unit-time COP, where  $\varepsilon_C$  is

the Carnot COP for reversible refrigerators. Yan and Chen [30] suggested to take  $\varepsilon Q_c/t_{\text{cycle}}$  as the target function, where  $t_{\text{cycle}}$  is the time for completing the whole Carnot-like cycle.  $\varepsilon$  and  $Q_c$  are respectively the COP of refrigerators and the heat absorbed by the working substance from the cold bath. They also optimized this target function and found the corresponding COP to be  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1$  for endoreversible refrigerators [30]. Recently, de Tomás *et al.* introduced a unified optimization  $\chi$ -criterion for heat devices which was defined as the product of the conversion efficiency and the heat absorbed per unit time by the working substance [31]. This  $\chi$ -criterion returns to the power output for heat engines while degenerates into the criterion proposed by Yan and Chen for refrigerators. Additionally, the COP at maximum  $\chi$  was also found to be  $\varepsilon_{CA} \equiv \sqrt{1 + \varepsilon_C} - 1$  for symmetric low-dissipation refrigerators [31]. Based on the work by de Tomás *et al.*, one of the present authors and his coworkers derived that the COP at maximum  $\chi$  was bounded between 0 and  $(\sqrt{9 + 8\varepsilon_C} - 3)/2$  for low-dissipation refrigerators [32]. The observed COP's for real refrigerators are also located in the region between these bounds, which is in good agreement with their theoretical estimation. These bounds were also confirmed by Izumida *et al.* with a minimally nonlinear irreversible model for refrigerators [33].

The above researches on refrigerators support that  $\chi$ -criterion is an appropriate figure of merit for refrigerators. The results obtained from maximizing this figure of merit for refrigerators have counterparts in those derived from maximizing the power output for heat engines. To begin with, the COP at maximum figure of merit ( $\varepsilon_{CA} \equiv \sqrt{1 + \varepsilon_C} - 1$ ) for the Yan-Chen endoreversible refrigerators corresponds to the EMP ( $\eta_{CA} \equiv 1 - \sqrt{1 - \eta_C}$ ) for the Curzon-Ahlborn endoreversible heat engines [30]. Next, the bounds of COP [0 and  $(\sqrt{9 + 8\varepsilon_C} - 3)/2$ ] at maximum figure of merit for low-dissipation refrigerators correspond to the bounds of efficiency [ $\eta_C/2$  and  $\eta_C/(2 - \eta_C)$ ] at maximum power for low-dissipation heat engines [32]. In particular, the COP at maximum fig-

ure of merit ( $\varepsilon_{CA} \equiv \sqrt{1 + \varepsilon_C} - 1$ ) for symmetric low-dissipation refrigerators also corresponds to the EMP ( $\eta_{CA} \equiv 1 - \sqrt{1 - \eta_C}$ ) for symmetric low-dissipation heat engines [31]. However, it is still lack of the counterpart of universal EMP for tight-coupling heat engines in the COP at maximum figure of merit for tight-coupling refrigerators when they operate between two baths at small temperature difference. Our main goal in this work is to complement this theoretical imperfection. We optimize the figure of merit for the Feynman ratchet as a typical tight-coupling refrigerator operating between two baths and then find that its COP at maximum figure of merit approaches to  $\sqrt{\varepsilon_C}$  (Note that  $\varepsilon_C^{-1}$  can also be understood as the relative temperature difference) when it operates between two baths at small temperature difference. It is obvious to see that this behavior is also shared by  $\varepsilon_{CA} \equiv \sqrt{1 + \varepsilon_C} - 1$ , which implies that  $\sqrt{\varepsilon_C}$  might be the universal COP at maximum figure of merit for tight-coupling refrigerators working between two baths at small temperature difference. This proposition is proved to be true under certain symmetric conditions within the framework of linear irreversible thermodynamics by adopting the new convention on the thermodynamic flux related to the heat transfer between two baths.

*Feynman ratchet as a refrigerator.*—The Feynman ratchet can be simplified as a Brownian particle walking in a periodic lattice labeled by  $\Theta_n$ , ( $n = \dots, -2, -1, 0, 1, 2, \dots$ ) with the fixed step size  $\theta$ . The ratchet potential is schematically depicted in Fig. 1, where the energy scale and the position of potential barrier are respectively denoted by  $\epsilon$  and  $\delta\theta$ . The parameter  $\delta$ , so called load distribution factor, takes value in between 0 and 1. The Brownian particle is in contact with a cold bath at temperature  $T_c$  in the left side of each potential barrier while it is in contact with a hot bath at temperature  $T_h$  ( $> T_c$ ) in the right side of each barrier. The particle is pulled from the left to the right side by a moment  $z$  due to the external force. In steady state, the forward and backward jumping rates can be respectively expressed as  $\omega_+ = k_0 e^{-(\epsilon - z\delta\theta)/T_c}$  and  $\omega_- = k_0 e^{-(\epsilon - z\delta\theta + z\theta)/T_h}$  according to the Arrhenius law [34]. In the expressions of jumping rates, the Boltzmann factor is set to 1 while  $k_0$  represents the bare rate constant with dimension of time<sup>-1</sup>. For simplicity, we introduce two abbreviated notations  $q \equiv \epsilon - z\delta\theta$  and  $w \equiv z\theta$ .

For the relative large load  $z$ , the forward jumping rate can be larger than the backward one. In this case, the net current

$$J \equiv \omega_+ - \omega_- = k_0 \left[ e^{-q/T_c} - e^{-(q+w)/T_h} \right] \quad (1)$$

is positive. In each forward step, the particle absorbs heat  $q \equiv \epsilon - z\delta\theta$  from the cold bath. Combining the input work  $w \equiv z\theta$  done by the external load, the absorbed heat will be released into the hot bath when the particle jumps over the barrier. Thus the total heat  $q + w$  will be

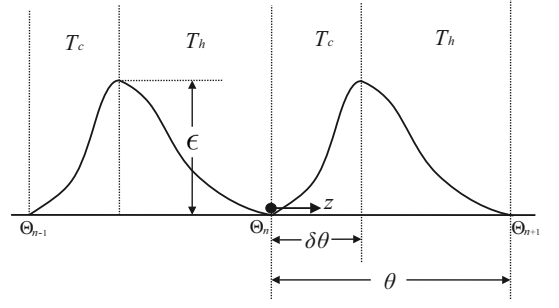


FIG. 1. Schematic diagram of Feynman ratchet as a refrigerator.

released into the hot bath in each forward step. The energy conversion in each backward step is exactly opposite of that in forward step mentioned above. Thus the net power input can be expressed as  $P = wJ$  while the heat absorbed from the cold bath or released into the hot bath per unit time can be expressed  $\dot{Q}_c = qJ$  or  $\dot{Q}_h = (q+w)J$ , respectively. Obviously, when  $J > 0$  the heat flows from the cold bath to the hot one, and the power input (i.e., the mechanical flux) is proportional to the thermal fluxes ( $\dot{Q}_c$  and  $\dot{Q}_h$ ). It is in this sense that the Feynman ratchet is regarded as a tight-coupling refrigerator.

The COP of this tight-coupling refrigerator can be expressed as

$$\varepsilon \equiv \dot{Q}_c / P = q / w. \quad (2)$$

Simultaneously, the figure of merit can be expressed as

$$\chi \equiv \varepsilon \dot{Q}_c = \frac{k_0 q^2}{w} \left[ e^{-q/T_c} - e^{-(q+w)/T_h} \right]. \quad (3)$$

Maximizing  $\chi$  with respect to the internal barrier height  $\epsilon$  and the external load  $z$ , we can obtain

$$(2 - q/T_c)e^{-q/T_c} = (2 - q/T_h)e^{-(q+w)/T_h}, \quad (4)$$

$$e^{-q/T_c} = (1 + w/T_h)e^{-(q+w)/T_h}. \quad (5)$$

Combining Eq. (2) and the above two equations, we derive that the COP at maximum figure of merit ( $\varepsilon_*$ ) satisfies the following transcendental equation:

$$\frac{\varepsilon_C - \varepsilon_*}{\varepsilon_C + 1} \left( \frac{2}{\varepsilon_*} - \frac{1}{\varepsilon_C} \right) = \ln \frac{(2 + \varepsilon_*)\varepsilon_C}{\varepsilon_*(\varepsilon_C + 1)}, \quad (6)$$

where  $\varepsilon_C = T_c/(T_h - T_c)$  is the COP of Carnot refrigerators. It is interesting and surprising that  $\varepsilon_*$  depends merely on  $\varepsilon_C$  (or equivalently speaking, the relative temperature difference between two baths) rather than the load distribution factor  $\delta$  although the expressions of jumping rates contain  $\delta$ .

It is almost impossible to achieve the analytic solution to Eq. (6). We will investigate the asymptotic behaviors at large temperature difference ( $\varepsilon_C \rightarrow 0$ ) and small temperature difference ( $\varepsilon_C \rightarrow \infty$ ), respectively. For the former case, considering  $0 \leq \varepsilon_* \leq \varepsilon_C$ , we transform Eq. (6)

into  $2\varepsilon_C/\varepsilon_* + \varepsilon_*/\varepsilon_C - 3 = \ln(2\varepsilon_C/\varepsilon_*)$ , from which we obtain

$$\varepsilon_* \simeq 0.524\varepsilon_C \quad (7)$$

when  $\varepsilon_C \rightarrow 0$ . This behavior is different from that of  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1$  which gives  $\varepsilon_{CA} = 0.5\varepsilon_C$  when  $\varepsilon_C \rightarrow 0$ . Thus there is no universal behavior of COP at maximum figure of merit for refrigerators at large temperature difference.

On the other hand, it is not hard to prove  $\varepsilon_* \rightarrow \infty$  and  $\varepsilon_*/\varepsilon_C \rightarrow 0$  when the temperature difference between two baths is very small (i.e.,  $\varepsilon_C \rightarrow \infty$ ) [35]. If multiplying  $1 + \varepsilon_C^{-1}$  on both sides of Eq. (6) and then expanding it into a series of  $\varepsilon_*^{-1}$  and  $\varepsilon_C^{-1}$ , we can derive

$$2/\varepsilon_C - \varepsilon_*/\varepsilon_C^2 + 2/\varepsilon_*\varepsilon_C - 2/\varepsilon_*^2 = 0, \quad (8)$$

when we neglect the contribution of higher order terms. The terms  $\varepsilon_*/\varepsilon_C^2$  and  $2/\varepsilon_*\varepsilon_C$  are of higher order relative to  $1/\varepsilon_C$  since  $\varepsilon_* \rightarrow \infty$  and  $\varepsilon_*/\varepsilon_C \rightarrow 0$  when  $\varepsilon_C \rightarrow \infty$ , thus Eq. (8) is further simplified into  $2/\varepsilon_C - 2/\varepsilon_*^2 \simeq 0$  when  $\varepsilon_C \rightarrow \infty$ . Its solution is

$$\varepsilon_* \simeq \sqrt{\varepsilon_C}, \quad (9)$$

which gives the asymptotic behavior of COP at maximum figure of merit for the Feynman ratchet as a refrigerator operating between two heat baths at small temperature difference.

We suggest to use an interpolation formula

$$\varepsilon_* = \sqrt{\varepsilon_C + \alpha^2} - \alpha \quad (10)$$

with  $\alpha = 1/(2 \times 0.524) = 0.954$  as the approximate solution to Eq. (6). It is easy to see that this formula degenerates into Eq. (7) and Eq. (9) when  $\varepsilon_C \rightarrow 0$  and  $\infty$ , respectively. We compare this interpolation formula with the numerical solution to Eq. (6) in Fig. 2. Surprisingly, this interpolation formula (solid line) does extremely approach to the numerical solutions (squares) to Eq. (6) obtained from the high precision computation when  $\varepsilon_C$  takes values in a relatively large range. This formula fits the numerical data better than formulas (7) and (9) depicted respectively as the dot line and dash dot line in Fig. 2. There exists a little difference between this interpolation formula and  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1$ . Both of them are respectively depicted as the solid line and dash line in the inset of Fig. 2, which reveals that the interpolation formula is much closer to the numerical solutions than  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1$ . The small relative error ( $< 0.8\%$ ) between interpolation formula (10) and the genuinely exact solution to Eq. (6) is quantitatively estimated in Ref. [35].

*Universality.*—It is obvious that  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1 \simeq \sqrt{\varepsilon_C}$  when  $\varepsilon_C \rightarrow \infty$ . That is, the Yan-Chen endoreversible refrigerators or the symmetric low-dissipation refrigerators also share the same asymptotic behavior (9)

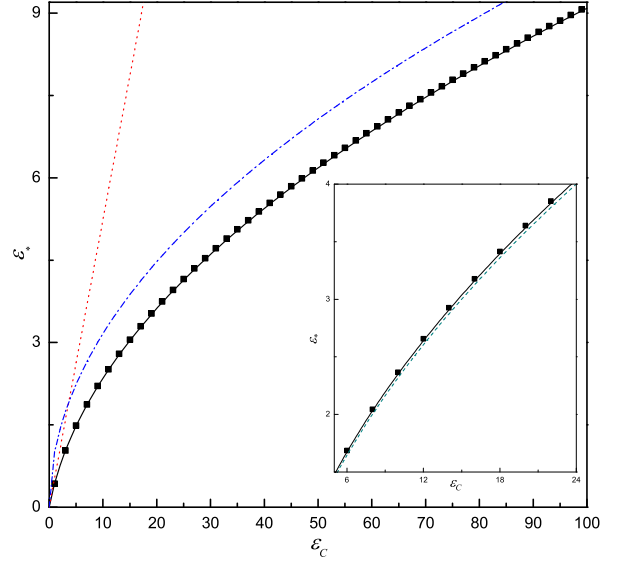


FIG. 2. (Color online) COP at maximum figure of merit for the Feynman ratchet as a refrigerator. The numerical solutions to Eq. (6) and interpolation formula (10) are depicted as the squares and solid line, respectively. The diagrams of functions (7) and (9) are depicted as the dot line and dash dot line, respectively. Inset graph shows the diagram of interpolation formula (solid line) and that of  $\varepsilon_{CA} \equiv \sqrt{\varepsilon_C + 1} - 1$  (dash line) as well as the numerical solutions (squares) in the range of  $5 < \varepsilon_C < 24$ .

as the Feynman ratchet at small temperature difference. Since all of them can be regarded as tight-coupling refrigerators, we may conjecture that a universal COP at maximum figure of merit  $\varepsilon_* \simeq \sqrt{\varepsilon_C}$  exists for tight-coupling refrigerators working between two baths at small temperature difference.

As it was done by Van den Broeck [6] for heat engines, here we consider a generic setup for a tight-coupling refrigerator shown in Fig. 3. An external force  $F$  is applied on the system and inputs a power  $P = F\dot{x}$  into the system, where  $x$  is the thermodynamically conjugate variable of  $F$ . The dot represents the derivative with respect to time. The corresponding thermodynamic force is  $X_1 = F/T$ , where  $T$  is the temperature of the system which can be well defined due to the assumption of local equilibrium. The thermodynamic flux conjugated to  $X_1$  is  $J_1 = \dot{x}$ . Then the power input can be expressed as  $P = J_1 X_1 T$  in terms of the thermodynamic flux and force. Assume that the system is in contact with a cold bath at temperature  $T_c \equiv T - s_c \Delta T$  and a hot bath at temperature  $T_h \equiv T + s_h \Delta T$  with  $\Delta T \ll T$ . The positive parameters  $s_c$  and  $s_h$  should satisfy  $s_c + s_h = 1$  due to the constraint  $\Delta T = T_h - T_c$ . Their specific values depend respectively on the coupling strengths between the model system and the cold or hot baths.

In steady state, the entropy production rate can be expressed as  $\sigma = \dot{Q}_h/T_h - \dot{Q}_c/T_c$  which can be further ex-

pressed as  $\sigma = (\dot{Q}_c + P)/T_h - \dot{Q}_c/T_c = P/T_h + \dot{Q}_c(1/T_h - 1/T_c)$  or  $\sigma = \dot{Q}_h/T_h - (\dot{Q}_h - P)/T_c = P/T_c + \dot{Q}_h(1/T_h - 1/T_c)$  due to the conservation of energy ( $\dot{Q}_c + P = \dot{Q}_h$ ). Both equations can be respectively expressed as  $\sigma = (P/T)[1 - s_h\Delta T/T + \mathcal{O}(\Delta T/T)^2] + \dot{Q}_c(1/T_h - 1/T_c)$  and  $\sigma = (P/T)[1 + s_c\Delta T/T + \mathcal{O}(\Delta T/T)^2] + \dot{Q}_h(1/T_h - 1/T_c)$  with the consideration of  $T_h \equiv T + s_h\Delta T$  and  $T_c \equiv T - s_c\Delta T$ , where  $\mathcal{O}(\Delta T/T)^2$  represents the term in the same order of  $(\Delta T/T)^2$ . Since  $s_c + s_h = 1$ , we can further derive  $\sigma = s_c\sigma + s_h\sigma = (P/T)[1 + \mathcal{O}(\Delta T/T)^2] + (s_c\dot{Q}_c + s_h\dot{Q}_h)(1/T_h - 1/T_c)$  which enlightens us to take  $X_2 \equiv 1/T_h - 1/T_c$  and

$$J_2 \equiv s_c\dot{Q}_c + s_h\dot{Q}_h \quad (11)$$

as the thermodynamic force and flux related to the heat transfer between two heat baths, respectively. The truncation error with the consideration of new convention (11) is in the order of  $(\Delta T/T)^2$  when the entropy production rate is expressed as  $\sigma = J_1X_1 + J_2X_2$ . The other conventions such as  $J_2 \equiv \dot{Q}_c$  or  $J_2 \equiv \dot{Q}_h$  result in the lower accuracy with the truncation error in the order of  $\Delta T/T$  when the entropy production rate is expressed as  $\sigma = J_1X_1 + J_2X_2$ . Therefore, we will adopt the high-precision convention (11) to deal with the tight-coupling refrigerator. With the consideration of Eq. (11) and  $\dot{Q}_c = \dot{Q}_h - P$ , the heat absorbed from the cold bath can be expressed as  $\dot{Q}_c = J_2 - s_hP$ .

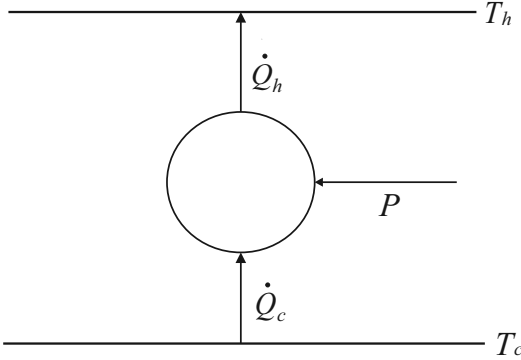


FIG. 3. Generic setup for a tight-coupling refrigerator.

According to linear irreversible thermodynamics, we write the linear relationship between the thermodynamic fluxes and forces:

$$J_1 = L_{11}X_1 + L_{12}X_2, \quad J_2 = L_{21}X_1 + L_{22}X_2, \quad (12)$$

where the Onsager coefficients satisfy  $L_{11} \geq 0$ ,  $L_{22} \geq 0$ ,  $L_{11}L_{22} - L_{12}L_{21} \geq 0$  and  $L_{12} = L_{21}$ . Furthermore, the tight-coupling condition  $L_{12}^2 = L_{21}^2 = L_{11}L_{22}$  leads to  $J_2/J_1 = L_{12}/L_{11}$ . With the consideration of this equation and  $P = TJ_1X_1$ , the COP can be expressed as  $\varepsilon \equiv \dot{Q}_c/P = L_{21}/TL_{11}X_1 - s_h$ , from which we have  $X_1 = L_{12}/TL_{11}(\varepsilon + s_h)$ . Substituting this equation into

the definition of figure of merit, we obtain

$$\chi \equiv \varepsilon\dot{Q}_c = L_{22}\varepsilon^2[1 + TX_2(\varepsilon + s_h)]/T(\varepsilon + s_h)^2. \quad (13)$$

Maximizing  $\chi$  with respect to  $\varepsilon$  (equivalently to  $X_1$ ), we find that the COP at maximum figure of merit satisfies  $\varepsilon_*^2 + 3s_h\varepsilon_* + 2s_h^2 + 2s_h/TX_2 = 0$  which gives  $\varepsilon_* = \sqrt{s_h^2/4 - 2s_h/TX_2} - 3s_h/2$ . With the consideration of  $X_2 \equiv 1/T_h - 1/T_c$  and  $\varepsilon_C \equiv T_c/(T_h - T_c)$ , we achieve

$$\varepsilon_* = \sqrt{s_h^2/4 + 2s_h\varepsilon_C(\varepsilon_C + 1)/(\varepsilon_C + s_c)} - 3s_h/2. \quad (14)$$

If the model system is symmetrically coupled with both heat baths such that  $s_h = s_c = 1/2$ , we derive  $\varepsilon_* \simeq \sqrt{\varepsilon_C}$  when  $\varepsilon_C \rightarrow \infty$  from Eq. (14). Therefore, we have proved the conjecture on the universal COP at maximum figure of merit for tight-coupling refrigerators symmetrically interacting with two heat baths at small temperature difference. In addition, in the extremely asymmetric cases of  $s_h = 0$  ( $s_c = 1$ ) and  $s_h = 1$  ( $s_c = 0$ ), equation (14) leads to  $\varepsilon_0 \equiv 0$  and  $\varepsilon_1 \equiv (\sqrt{9 + 8\varepsilon_C} - 3)/2$ , respectively, which surprisingly equal to the global lower and upper bounds  $[\varepsilon_- \equiv 0 \text{ and } \varepsilon_+ \equiv (\sqrt{9 + 8\varepsilon_C} - 3)/2]$  of COP at maximum figure of merit for low-dissipation refrigerators which are also found to be reached in the case of extremely asymmetric dissipations [32].

*Conclusion and discussion.*— In the above discussion, we investigate the COP at maximum figure of merit for the Feynman ratchet as a refrigerator and find that the corresponding COP can be approximately expressed as interpolation formula (10). In the limit of small temperature difference between two baths, this formula and  $\varepsilon_{CA} \equiv \sqrt{1 + \varepsilon_C} - 1$  share the same asymptotic behavior (9). This universal asymptotic behavior is proved to be available for refrigerators tightly and symmetrically coupled with two baths at small temperature difference.

The key point in our proof of universality is to adopt new convention (11) as the thermodynamic flux related to the heat transfer between two baths. If we adopt this convention and recalculate the model for a tight-coupling heat engine in Ref. [6], we still obtain the same result of universal EMP ( $\eta_C/2$ ) as Van den Broeck did. Interestingly, if further considering the symmetric-coupling between the engine and its two baths, we can return to the universal EMP ( $\eta_C/2 + \eta_C^2/8$  proved by Esposito *et al.* [10]) up to the second order term [35]. In this sense, asymptotic behavior (9) for refrigerators tightly and symmetrically coupled with two baths at small temperature difference can be regarded as the counterpart of universal EMP for tight-coupling heat engines operating between two baths at small temperature difference in the presence of left-right symmetry. In addition, we find the same surprise that the global lower and upper bounds of EMP for low-dissipation heat engines obtained in Ref. [13] can be achieved when we adopt convention (11) to deal with the tight-coupling heat engine in the extremely asymmetric

cases of  $s_c = 0$  ( $s_l = 1$ ) and  $s_c = 1$  ( $s_h = 0$ ) [35]. It is a fantastic fact that a local theory valid for the small temperature difference can give the correctly global bounds available for the arbitrary temperature difference. In our opinion, it is a challenge to investigate whether this fact takes place merely by coincidence or due to some underlying reasons.

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- [1] F. L. Curzon and B. Ahlborn, Am. J. Phys. **43**, 22 (1975).
- [2] L. Chen and Z. Yan, J. Chem. Phys. **90**, 3740 (1989).
- [3] J. Chen, J. Phys. D: Appl. Phys. **27**, 1144 (1994).
- [4] A. Bejan, J. Appl. Phys. **79**, 1191 (1996).
- [5] L. Chen, C. Wu, and F. Sun, J. Non-Equil. Thermody. **24**, 327 (1999).
- [6] C. Van den Broeck, Phys. Rev. Lett. **95**, 190602 (2005).
- [7] B. Jiménez de Cisneros and A. Calvo Hernández, Phys. Rev. Lett. **98**, 130602 (2007).
- [8] T. Schmiedl and U. Seifert, Europhys. Lett. **81**, 20003 (2008).
- [9] Z. C. Tu, J. Phys. A: Math. Theor. **41**, 312003 (2008).
- [10] M. Esposito, K. Lindenberg, and C. Van den Broeck, Phys. Rev. Lett. **102**, 130602 (2009).
- [11] Y. Apertet, H. Ouerdane, C. Goupil, and P. Lecoeur, Phys. Rev. E **85**, 041144 (2012).
- [12] U. Seifert, Rep. Prog. Phys. **75**, 126001, 2012 (2012).
- [13] M. Esposito, R. Kawai, K. Lindenberg, and C. Van den Broeck, Phys. Rev. Lett. **105**, 150603 (2010).
- [14] B. Gaveau, M. Moreau and L. S. Schulman, Phys. Rev. Lett. **105**, 060601 (2010).
- [15] Y. Wang and Z. C. Tu, Phys. Rev. E **85**, 011127 (2012).
- [16] Y. Wang and Z. C. Tu, Europhys. Lett. **98**, 40001 (2012).
- [17] Y. Izumida and K. Okuda, Europhys. Lett. **97**, 10004 (2012).
- [18] J. Wang, J. He and Z. Wu, Phys. Rev. E **85**, 031145 (2012).
- [19] J. Wang and J. He, Phys. Rev. E **86**, 051112 (2012).
- [20] Y. Wang and Z. C. Tu, Commun. Theor. Phys. in press (2013); see also e-print arXiv:1201.0848.
- [21] L. Chen, F. Sun, and W. Chen, Energy **20**, 1049 (1995).
- [22] L. Chen, Z. Ding, and F. Sun, Appl. Math. Model. **35**, 2945 (2011).
- [23] J. Chen and Z. Yan, J. Appl. Phys. **84**, 1791 (1998).
- [24] B. Lin and J. Chen, J. Phys. A: Math. Theor. **42**, 075006 (2009).
- [25] J. He, J. Chen and B. Hua, Phys. Rev. E **65**, 036145 (2002).
- [26] B. Jiménez de Cisneros, L. A. Arias-Hernández, and A. Calvo Hernández, Phys. Rev. E **73**, 057103 (2006).
- [27] C. de Tomas, J. M. M. Roco, A. Calvo Hernández, Yang Wang, and Z. C. Tu, Phys. Rev. E **87**, 012105 (2013).
- [28] A. E. Allahverdyan, K. Hovhannisyan and G. Mahler Phys. Rev. E **81**, 051129 (2010).
- [29] S. Velasco, J. M. M. Roco, A. Medina, and A. Calvo Hernández, Phys. Rev. Lett. **78**, 3241 (1997).
- [30] Z. Yan and J. Chen, J. Phys. D: Appl. Phys. **23**, 136 (1990).
- [31] C. de Tomás, A. Calvo Hernández and J. M. M. Roco, Phys. Rev. E **85**, 010104(R) (2012).
- [32] Y. Wang, M. Li, Z. C. Tu, A. Calvo Hernández, and J. M. M. Roco, Phys. Rev. E **86**, 011127 (2012).
- [33] Y. Izumida, K. Okuda, A. Calvo Hernández and J. M. M. Roco, Europhys. Lett. in press.
- [34] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* vol 1 (Addison-Wesley, Reading Mass., 1966)
- [35] See Supplemental Material at [URL will be inserted by publisher] for investigating the COP at maximum figure of merit for the Feynman ratchet, estimating error, explaining the logic of new convention, and reconsidering heat engines with the new convention.